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Continuously Measured Systems, Path Integrals, and Information

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A continuously measured quantum system may be described by restricted path integrals (RPI) or equivalently by non-Hermitian Hamiltonians. The measured system is then considered as an open system, the influence of the environment being taken into account by restricting the path integral or by inclusion of an imaginary part in the Hamiltonian. This way of description of measurements naturally follows from the Feynman form of quantum mechanics without any additional postulates and may be interpreted as an information approach to continuous quantum measurements. This reveals deep features of quantum physics concerning relations between quantum world and its classical appearance.

1. INTRODUCTION

It is widely believed that quantum mechanics is not closed, and that only after adding some form of quantum theory of measurement does it become a complete and self-sufficient theory. We shall argue that the theory of continuous quantum measurements may in fact be considered as a natural part of quantum mechanics provided the latter is taken in the Feynman path-integral form (Feynman, 1948) including the rules for summing up probability amplitudes. The main instruments of the resulting theory of continuous measurements are restricted path integrals (RPI) and non-Hermitian Hamiltonians (Mensky, 1979a, b, 1993). The RPI approach may be regarded as an information approach to continuous quantum measurements just as the von Neumann's projection postulate presents an information approach to instantaneous quantum measurements.

So-called "instantaneous" measurements (which are in reality not instantaneous, but very short) may be obtained as a limiting case of continuous

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measurements. Therefore, the whole quantum theory of measurements may be derived, in the framework of the RPI approach, from quantum mechanics in the path-integral form. Hence, quantum mechanics may be considered as a closed theory. It looks nonclosed only if the overidealized concept of an instantaneous measurement is considered instead of realistic concept of a continuous measurement.

2. CONTINUOUS QUANTUM MEASUREMENTS AND RESTRICTED PATH INTEGRALS (RPI)

During recent decades the theory of continuous quantum measurements has been under thorough investigation (Mensky, 1979a, b, 1993, Zeh, 1971; Davies, 1976; Srinivas, 1977; Walls and Milburn, 1985; Milburn, 1988; Joos and Zeh, 1985; Diosi, 1988; Peres, 1993; Carmichael, 1993). The interest in this field significantly increased in connection with the quantum Zeno effect predicted in Misra and Sudarshan (1977), Chiu *et al.* (1977), and Peres (1980) and experimentally verified in Itano *et al.* (1990). In most cases studying continuous quantum measurements was based on particular models of measuring devices. In contrast, the phenomenological and therefore model-independent restricted-path-integrals (RPI) approach to continuous measurements has been proposed (Mensky, 1979a, b, 1983, 1993; see also Khalili, 1981; Barchielli *et al.*, 1982; Caves, 1986, 1987) following the idea of Feynman (1949).

The measured system is considered in the RPI theory of measurements as an open system. The back influence of the measuring device (environment) onto the measured system is taken into account by restricting the path integral. The restriction is determined by the information which the measurement supplies about the measured system. Let us consider the main points of this approach.

The evolution of a closed quantum system during a time interval T is described by the evolution operator U_T. The matrix element of the operator U_T between the states with definite positions is called the propagator and may be expressed in the form of the Feynman path integral[3]

$$U_T(q'', q') = \langle q''|U_T|q'\rangle = \int d[p]\, d[q] \exp\left[\frac{i}{\hbar}\int_0^T (p\dot{q} - H(p, q, t))\right] \quad (1)$$

If the system with the same dynamical properties (the same Hamiltonian) undergoes a continuous (prolonged in time) measurement (and therefore is considered as being open, interacting with a measuring device or environ-

[3] It is convenient for our goals to use a phase-space representation of the path integral. The variables q and p may be multidimensional.

ment), its evolution may be described (Mensky, 1993) by the set of *partial evolution operators* U_T^α depending on the output (readout) α of the measurement[4]

$$|\psi_T^\alpha\rangle = U_T^\alpha|\psi_0\rangle, \qquad \rho_T^\alpha = U_T^\alpha\rho_0(U_T^\alpha)^\dagger$$

The partial propagators are expressed by restricted path integrals. This means (Mensky, 1993) that the path integral for U_T^α must be of the form (1), but restricted according to the information given by the measurement readout α. The information given by α may be described by a weight functional $w_\alpha[p, q]$ (positive, with values between 0 and 1) so that the partial propagator has to be written as a weighted path integral

$$\begin{aligned} U_T^\alpha(q'', q') &= \langle q''|U_T^\alpha|q'\rangle \\ &= \int d[p]\, d[q]\, w_\alpha[p, q] \exp\left[\frac{i}{\hbar}\int_0^T (p\dot{q} - H(p, q, t))\right] \end{aligned} \tag{2}$$

The probability density for each α to arise as a measurement readout is given (Mensky, 1993) by the trace of the density matrix ρ_T^α, so that the probability for α to belong to some set $\mathcal{A}$ of readouts is equal to

$$\mathrm{Prob}(\alpha \in \mathcal{A}) = \int_{\mathcal{A}} d\alpha\, \mathrm{Tr}\, \rho_T^\alpha \tag{3}$$

with an appropriate measure $d\alpha$ on the set of readouts.

All this concerns the situation when the measurement readout α is known (selective description of the measurement). If the readout is unknown (nonselective description), the evolution of the measured system may be represented by the density matrix

$$\rho_T = \int d\alpha\, p_T^\alpha = \int d\alpha\, U_T^\alpha\rho_0(U_T^\alpha)^\dagger \tag{4}$$

The generalized unitarity condition

$$\int d\alpha (U_T^\alpha)^\dagger U_T^\alpha = \mathbf{1}$$

provides conservation of probabilities.

In the special case when monitoring an observable $A = A(p, q, t)$ is considered as a continuous measurement, the measurement readout is given by the curve

$$[a] = \{a(t)|0 \le t \le T\}$$

[4] Physically the readout is recorded in some way or another in the state of the environment (measuring device).

characterizing the values of this observable in different time moments. If the square average deflection

$$\langle (A-a)^2 \rangle_T = \frac{1}{T}\int_0^T [A(t) - a(t)]^2\, dt$$

is taken as a measure of deviation of the observable $A(t) = A(p(t), q(t), t)$ from the readout $a(t)$, then the weight functional describing the measurement may be taken[5] in the Gaussian form:

$$w_{[a]}[p, q] = \exp\left\{-\kappa \int_0^T [A(t) - a(t)]^2\, dt\right\}$$

The constant κ characterizes the resolution of the measurement and may be expressed in terms of the "measurement error" Δa_T which is achieved during the period T of the measurement:

$$\kappa = \frac{1}{T\Delta a_T^2} \quad \text{is constant,} \qquad \text{hence} \quad \Delta a_T^2 \sim \frac{1}{T}$$

The resulting path integral

$$U_T^{[a]}(q'', q') = \int d[p]\, d[q] \times \exp\left\{\frac{i}{\hbar}\int_0^T (p\dot{q} - H)\, dt - \kappa \int_0^T (A(p, q, t) - a(t))^2\, dt\right\}$$

has the form of a conventional (nonrestricted) Feynman path integral (1) but with the *effective Hamiltonian*

$$H_{[a]}(p, q, t) = H(p, q, t) - i\kappa\hbar(A(p, q, t) - a(t))^2 \tag{5}$$

instead of the original Hamiltonian H. Therefore, instead of calculating a restricted path integral, one may solve the Schrödinger equation with a non-Hermitian effective Hamiltonian:

$$\frac{\partial}{\partial t}|\psi_i\rangle = \left(-\frac{i}{\hbar} H - \kappa(A - a(t))^2\right)|\psi_i\rangle \tag{6}$$

If we solve this equation with the initial wave function ψ_0 describing the initial state of the measured system, then the solution ψ_T in the final time moment represents the state of the system after the measurement, under the condition that the measurement results in the readout $[a]$. The wave function

[5] The choice of the weight functional determines the class of measurements under consideration.

ψ_T obtained in this way has a nonunit norm. If the initial wave function is normalized, then the norm of the final wave function, according to equation (3), determines the probability density of the measurement output $[a]$:

$$P[a] = \|\psi_T\|^2 \tag{7}$$

We obtain the following scheme of calculation for the *selective description* of the continuous measurement (when the readout is known):

(1) Choose an arbitrary readout $[a]$ and solve equation (6).
(2) The probability density of $[a]$ is given by equation (7).
(3) The state of the system after the measurement is $|\psi_T\rangle$.

The *nonselective description* of the measurement (if the readout is unknown) is given by the density matrix σ_i defined by (4) and satisfying (Mensky, 1994) the equation

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] - \frac{\kappa}{2}[A, [A, \rho]] \tag{8}$$

3. RPI AS AN INFORMATION APPROACH TO CONTINUOUS QUANTUM MEASUREMENTS

The description of continuous quantum measurements by restricted path integrals (RPI) may be justified in different ways. The most direct way (Konetchnyi *et al.*, 1993) is an analysis of a composite system containing both the measured system and its environment (measuring device). Alternatively, one can consider a series of instantaneous measurements with the help of von Neumann's projection postulate and then go over to the continuous measurement as a limit of small time intervals between the instantaneous measurements (Mensky, 1983, 1993).

It is very interesting, however, that in the framework of the Feynman version of quantum mechanics the RPI approach needs no justification at all. This approach is natural and self-consistent in this framework. This is why Feynman was able to formulate the idea of the RPI approach as a short remark in his paper (Feynman, 1949). Moreover, the RPI approach, naturally following from Feynman quantum mechanics, is more general than what can be obtained in the limit of a series of instantaneous measurements. It describes a wider class of continuous and continual (protracted in time and space) measurements than those derivable as limits of some or other repeated measurements (Mensky, 1993).

The reason the RPI approach follows from the path-integral version of quantum mechanics is that the concept of *probability amplitude* is used in a much more comprehensive way in this version. In particular, the amplitude

$$A[p, q] = \exp\left[\frac{i}{\hbar}\int_0^T (p\dot{q} - H(p, q, t))\right] \tag{9}$$

is introduced and physically interpreted as a probability amplitude for the system to propagate along a definite path in the phase space. If this is accepted, then the usual quantum mechanical rules for amplitudes determine the amplitudes for more complicated events, in particular, for propagation of the system between two points of the configuration space. If the system is closed and therefore nothing is known about the path along which it propagates, all amplitudes of the form (9) have to be summed up, leading to the conventional Feynman integral (1). If a continuous measurement takes place (so that the system is open), one has to keep in mind that the measurement supplies some information about the evolution of the system. In summing up the amplitudes (9) this information must be taken into account.

If the information given by the measurement can be expressed by a weight functional $w_\alpha[p, q]$, then summation of amplitudes takes the form of equation (2). Hence, instead of directly postulating partial propagators, we can *derive* them from the more basic postulates of the path-integral version of quantum mechanics.

This is both interesting and unexpected. It is commonly believed that quantum mechanics is not closed, since it does not include any theory of measurements. A theory of measurements (for example, von Neumann's projection postulate) is customarily appended as a necessary counterpart forming, together with quantum mechanics, a closed theory. However, this proves to be unnecessary. As argued above, the path-integral formulation of quantum mechanics includes also the RPI theory of continuous measurements. A theory of instantaneous measurements (including von Neumann's postulate) may be then obtained as a limit (Calarco, 1995).

We see therefore that the seeming necessity to postulate a theory of measurement independently of quantum mechanics is only a consequence of overidealization. The origin of this necessity is in treating a measurement as an instantaneous act. An instantaneous measurement appears to be external with respect to quantum mechanical (Schrödinger) evolution and to need special postulates. The situation, however, is radically different if the measurement is considered as a temporally extended process and quantum mechanics is accepted in the path-integral form. Then the measurement may be described in a nonseparably integral way with the quantum mechanical evolution (Mensky, 1988). The mathematical apparatus describing both counterparts of this unity is given by restricted path integrals.

Restricted path integrals describe the influence of the measuring environment on the open (measured) quantum system without an explicit model of the environment. Instead, the RPI approach needs only very general characteristics of the environment. Namely, it is necessary to know what information about the evolution of the system is recorded in the state of the environment (as a measurement readout). This information determines what weight functional has to be used in the path integral. Having a restricted path integral, we can describe correctly both the probability distribution of measurement readouts and the final state of the measured system.

Thus, the influence of the environment on the system of interest may be given in terms of information. Therefore, the RPI approach is in fact an *information approach* to the theory of continuous quantum measurements. This indicates the fundamental character of the approach.

The information approach in the quantum theory of measurement is not novel. The very first quantum theory of (instantaneous) measurements based on the von Neumann reduction postulate may be considered as an example of an information approach. In this theory both the probability distribution for measurement outputs and the final state of the system may be found if we know what information the measurement supplies. For example, for the measurement of an observable with a discrete spectrum we need to know what eigenvalue is obtained as the measurement output. Given this information, we can determine, with the help of the corresponding projector, both the probability of the measurement output and the final state of the system.

In the RPI theory of continuous quantum measurements the information principle obtains one more realization having a rich structure and wide range of applications. It has been shown above that the RPI theory is, in contrast to the von Neumann postulate, a natural part of (the path-integral version of) quantum mechanics.

4. CONCLUSION

The restricted-path-integral (RPI) approach to continuous quantum measurements enables one to describe the influence of the measurement on the measured system without explicitly considering any model of the measuring device (environment). Instead of the model, one needs to know the information supplied by the measurement.

It is remarkable that knowledge of this information is sufficient for correctly accounting for the influence of the measuring environment on the system. This feature enables one to derive the RPI theory of continuous measurements from the path-integral version of quantum mechanics without any additional postulates. Besides, this shows that the RPI approach is an information approach to continuous measurements just as the von Neumann projection theory is an information approach to instantaneous measurements.

The information character of the RPI theory indicates clearly that it reveals deep internal qualities of quantum mechanics. This theory may be used to investigate further the relations between quantum and classical physics, between the quantum world and its classical appearance.

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